Numerical integration

Use a midpoint method with $\Delta x = 0.1$ to estimate the following integrals

- $\int_0^2 x^3 dx.$
- $\int_2^4 x^3 dx.$
- $\int_0^\pi \sin x dx.$
- Repeat the above integrals with $\Delta x = 0.01$. Show that we get closer to the analytical solution.
- Calculate the integrals using the Simpson's method (scipy.integrate.simps).

Here is the code for the midpoint method, as described in the lecture:

```
import numpy
dx = 0.01
x = numpy.arange(0.0,2.0,dx)
x += dx/2
y = x**2
IntegralValue = numpy.sum(y*dx)
print('Numerical estimate of integral is %8.8g. Analytical solution is %8.8g.'\
%(IntegralValue,1.0/3.0*2.0**3))
```

Bonus exercise

Calculating numerical integrals involving ∞ can be tricky at first sight, because it is impossible to numerically create an array extending to ∞. In the following we show that doing a substituion will help us evaluating the following integral

$$\int_1^\infty \frac{1}{x^2} dx \tag{1}$$

- Do the substitution $u = \arctan x$, use the identity $du/dx = 1/(1 + x^2)$, and show that the integral can be rewritten as

$$\int_1^\infty \frac{1}{x^2} dx = \int_{\arctan(1)}^{\frac{\pi}{2}} \frac{1 + \tan^2(u)}{\tan^2(u)} du. \tag{2}$$

  ... this is a well-behaved integral without infinities.

- Evaluate the integral on right-hand-side of Eq. 2 numerically.
- Numerically evaluate $\int_1^\infty \frac{1}{x^3} dx$